3 MATRICES

Curriculum

Contents Included	Periods
a. Introduction of matrix, order and components	20
b. Types of matrix: row, column, zero, square, diagonal, scalar, unit, equal, symmetric and triangular matrices	
c. Operation of matrices (Addition and subtraction)	
d. Properties of matrix addition	
e. Transpose of a matrix	
f. Multiplication matrix - Scalar multiplication - Multiplication of matrices - Properties of matrix multiplication (closure, associative and distributive)	

Objectives

On completion of this unit, students will be able to:

- define matrix and state components of matrix.
- find the matrix with given formula for its elements and order.
- state the kinds of matrices and perform the addition and subtraction of matrices and also state the properties of addition of two matrices.
- find the transpose of the given matrix.
- find the matrix multiplication and test the properties of matrix multiplication.

Materials

We can represent different information about the data in the matrix form. It is very clear by representing the given information in the matrix form.

The following table shows the number of boy and girl students in three different classes in a school. The information can be displayed in the form of matrices. The first row represents the number of boys, the second row represents the number of girls. Similarly, the first column represents students of class VIII, the second column represents the students of class IX and the third column represents the students of class X.

Class	VIII	IX	X
Boys	15	18	16
Girls	18	15	13

$$\begin{array}{cc} \text{Class} & \begin{array}{ccc}\text{VIII} & \text{IX} & \text{X}\end{array}\\ \begin{array}{c}\text{Boys}\\ \text{Girls}\end{array} & \begin{pmatrix}15 & 18 & 16\\ 18 & 15 & 13\end{pmatrix}\end{array}$$

Specification Grid

Contents	Knowledge	Understanding	Application	Higher ability	Total Questions	Total Marks
	1 mark each	2 marks each	4 marks each	5 marks each		
Matrices	1	2	1	–	4	9

3.1 Representation and Types of Matrices

Introduction

A set of numbers arranged in a rectangular form in rows and columns and enclosed by round (or square) bracket is known as a matrix. The plural form of matrix is matrices. The theory of matrices has become one of the most important and powerful tool in Mathematics. It has wide applications in various fields such as Statistics, Engineering, Economics etc. The following table gives the number of students in three classes of two schools.

	Class X	Class XI	Class XII
School A	27	25	20
School B	34	32	31

The above information can be arranged in following rectangular form between round brackets.

$$\begin{array}{r c l}
 & \begin{array}{ccc} \text{X} & \text{XI} & \text{XII} \end{array} & \\
\begin{array}{r} \text{School A} \\ \text{School B} \end{array} & \begin{pmatrix} 27 & 25 & 20 \\ 34 & 32 & 31 \end{pmatrix} & \begin{array}{l} 1^{st}\text{ row} \\ 2^{nd}\text{ row} \end{array} \\
 & \begin{array}{ccc} 1^{st}\text{ col.} & 2^{nd}\text{ col.} & 3^{rd}\text{ col.} \end{array} &
\end{array}$$

The numbers 27, 25 and 20 are in the first horizontal line and are said to be in the first row (R_1) and the numbers 34, 32 and 31 are said to be in the second row (R_2). Again the numbers 27 and 34 are in the first vertical line and are said to be in the first column (C_1) and 25, 32 and 20, 31 are said to be in the second column (C_2) and the third column (C_3) respectively.

The first row shows the number of students in three different classes of school A and the first column shows the number of students in class X of school A and B and so on.

Order of matrix

Each number of a matrix is known as an element or entry or component of the matrix. The number of rows and columns taken together gives an order or size or dimension of a matrix. The matrices are denoted by capital letters and the elements are denoted by small letters.

For example, $A = \begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix}$

A is a matrix and a, b, c, d, e and f are its elements. The number of rows of matrix A = 2 and its number of columns = 3, so the order of matrix A is 2×3. A matrix 'A' of order 2×3 is denoted by $A_{2\times3}$.

The elements of a matrix are generally denoted by the corresponding small letters with double suffix where the first suffix indicates the number of rows and the second suffix indicates the number of columns of that element.

For example, $B = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & f_{23} \end{pmatrix}$

Here, b_{12} is an element belongs to the first row and the second column. Similarly, b_{13} is the element of the first row and third column.

Example 1:

If $M = \begin{pmatrix} 2 & 3 \\ 11 & -4 \\ -7 & 2 \end{pmatrix}$, *find*

a. *The order of M and*

b. *The values of* m_{21}, m_{12} *and* m_{32}.

Here, $M = \begin{pmatrix} 2 & 3 \\ 11 & -4 \\ -7 & 2 \end{pmatrix}$

a. Number of rows = 3 and number of columns = 2, the order of M is 3×2.

b. m_{21} = the element belongs to the 2nd row and the 1st column = 11.

m_{12} = the element belongs to the 1st row and the 2nd column = 3.

m_{32} = the element belongs to the 3rd row and the 2nd column = 2.

Types of Matrices

There are different types of matrices according to the dimensions of matrices and the nature of the elements.

a. Row Matrix or Row Vector

A matrix having only one row is known as a row matrix or a row vector.
For example, $A = (2 \quad 5)$, $B = (x \quad y \quad z)$.
A and B are row matrices of orders 1×2 and 1×3 respectively.

b. Column Matrix or Column Vector

A matrix having only one column is called a column matrix or column vector.

For example: $P = \begin{pmatrix} a \\ b \end{pmatrix}$, $Q = \begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix}$, P and Q are column matrices of orders 2×1 and 3×1 respectively.

c. Zero or Null Matrix

A matrix in which each of its element zero is called a zero matrix or null matrix.

For example, $M = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$, $N = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, M and N are null matrices of orders 2×2 and 2×3 respectively.

d. Rectangular Matrix

A matrix in which the number of rows and columns are different is called a rectangular matrix.

For example: $C = \begin{pmatrix} 2 & 4 & -2 \\ 1 & -5 & -1 \end{pmatrix}$, $D = \begin{pmatrix} 2 & 3 \\ 4 & 1 \\ -1 & 0 \end{pmatrix}$, C and D are rectangular matrices of orders 2×3 and 3×2 respectively.

e. Square Matrix

A matrix having equal number of rows and columns is called a square matrix. For example: $R=\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $Q=\begin{pmatrix} 1 & -1 & 7 \\ 2 & 2 & 8 \\ 3 & -4 & 0 \end{pmatrix}$ R and Q are square matrices of orders 2×2 and 3×3 respectively. A square matrix of order n×n is often said to be a square matrix of order n.

f. Diagonal Matrix

A square matrix having all non-diagonal elements zero is called a diagonal matrix. The elements belonging to same number of rows and columns are called the diagonal elements. a_{11}, a_{22}, a_{33} etc. are called the diagonal elements. For example: $D=\begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix}$, $E=\begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 5 \end{pmatrix}$

D and E are diagonal matrices of orders 2 and 3 respectively.

g. Scalar Matrix

A diagonal matrix with all its diagonal elements equal is called a scalar matrix.

$X=\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$, $Y=\begin{pmatrix} -4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -4 \end{pmatrix}$ X and Y are scalar matrices of orders 2 and 3 respectively.

h. Unit or Identity Matrix

A square matrix with all diagonal elements unity and the rest zero is called a unit matrix or an identity matrix. $T=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, $S=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ T and S are unit matrices of orders 3 and 2 respectively. A unit matrix of order 2 is denoted by I_2 and I_3 means a unit matrix of order 3.

i. Equal Matrices

Two matrices of the same order are said to be equal if their corresponding elements are equal.

For the matrices $P=\begin{pmatrix} 2 & -1 \\ 4 & 3 \end{pmatrix}$, $Q=\begin{pmatrix} 2 & -1 \\ 4 & 3 \end{pmatrix}$, $A=\begin{pmatrix} 1 & 3 & -2 \\ 4 & -5 & 0 \\ 3 & -1 & 6 \end{pmatrix}$, $B=\begin{pmatrix} 1 & 3 & -2 \\ 4 & -5 & 0 \\ 3 & -1 & 6 \end{pmatrix}$

P and Q are equal matrices as they are matrices of same order having corresponding equal elements. Similarly, A and B are equal matrices.

j. Symmetric Matrix

A square matrix is known as symmetric matrix if interchange of rows and columns makes no change in the given original matrix. For example,

$$P=\begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}, A=\begin{pmatrix} 1 & 2 & -3 \\ 2 & 4 & 5 \\ -3 & 5 & 6 \end{pmatrix}$$

i.e. having same elements in rows and columns of a square matrix is called symmetric matrix.

k. Triangular Matrix

A square matrix in which all the elements above or below (or both) the main diagonal are zero is called a triangular matrix. There are two types:

i. Upper Triangular Matrix

A square matrix in which all the elements below the main diagonal are zero is called an ***Upper Triangular*** Matrix. For example, $\begin{pmatrix} 1 & -5 \\ 0 & 6 \end{pmatrix}$ and $\begin{pmatrix} 1 & 2 & -3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{pmatrix}$ are upper triangular matrices.

ii. Lower Triangular Matrix

A square matrix in which all the elements above the main diagonal are zero is called a ***Lower Triangular*** Matrix. For example, $\begin{pmatrix} 5 & 0 \\ 2 & 3 \end{pmatrix}$ and $\begin{pmatrix} 3 & 0 & 0 \\ 5 & -4 & 0 \\ 6 & 6 & -6 \end{pmatrix}$ are lower triangular matrices.

Example 2:

Construct a 2 × 3 matrix whose elements are given by aij = 4i – j.

As, aij = 4i – j

when $i = 1, j = 1$: $a_{11} = 4 \times 1 - 1 = 3$; $i = 2, j = 1$: $a_{21} = 4 \times 2 - 1 = 7$

$i = 1, j = 2$: $a_{12} = 4 \times 1 - 2 = 2$; $i = 2, j = 2$: $a_{22} = 4 \times 2 - 2 = 6$

$i = 1, j = 3$: $a_{13} = 4 \times 1 - 3 = 1$; $i = 2, j = 3$: $a_{23} = 4 \times 2 - 3 = 5$

Hence, the required 2×3 matrix is

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix} = \begin{pmatrix} 3 & 2 & 1 \\ 7 & 6 & 5 \end{pmatrix}$$

Example 3:

If $\begin{pmatrix} x-1 & 3 \\ 5 & y \end{pmatrix} = \begin{pmatrix} -3 & x+z \\ w-y & 2 \end{pmatrix}$***, find the values of w, x, y and z.***

We have, $\begin{pmatrix} x-1 & 3 \\ 5 & y \end{pmatrix} = \begin{pmatrix} -3 & x+z \\ w-y & 2 \end{pmatrix}$

By the definition of equality of matrices,

$x - 1 = -3$

$\therefore \quad x = -2$

$x + z = 3$

or, $-2 + z = 3$

$\therefore \quad z = 5$

Again, $y = 2$ and $w - y = 5$

or, $\quad w - 2 = 5$

$\therefore \quad w = 7$

Hence, $w = 7$, $x = -2$, $y = 2$ and $z = 5$.

EXERCISE : 3.1

1. a. Write an example of row matrix of order 1×4.

   b. Write an example of column matrix of order 3×1.

2. a. If the matrix $A = \begin{pmatrix} x & 0 \\ 0 & x \end{pmatrix}$, then write its type.

   b. If the matrix $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$, then write its type.

3. a. Write the order of matrix $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$.

   b. In matrix $P = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix}$, write its order and type.

4. a. If matrix $A = \begin{pmatrix} 21 & 12 & 16 \\ 24 & 13 & 17 \end{pmatrix}$ and $a_{2j} = 17$, find the value of j.

   b. If the matrix $A\,(a_{ij})_{2\times3}$, where $a_{ij} = i + j$, then what is the value of a_{21}? Write it.

5. a. If matrix $A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$, then write the number of elements of A.

   b. If $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ x & y \end{pmatrix}$, then find the value of x and y.

6. a. A matrix has 8 elements. What are the possible orders can it have?

   b. If A is a matrix of order 2×5, find the number of elements of A.

7. What are the orders of the following matrices:

   a. $\begin{pmatrix} 1 & 3 & 4 \\ 8 & -2 & 2 \end{pmatrix}$ b. $\begin{pmatrix} a & b \\ -p & q \\ m & -n \end{pmatrix}$ c. $\begin{pmatrix} 3 & 1 & 2 \\ 4 & -2 & 7 \\ 0 & -4 & 3 \\ 9 & 6 & 1 \end{pmatrix}$

8. a. A school has three branches A, B and C. The number of boys and girls in those branches are as follows:

   Branch A has 270 boys and 209 girls,

   Branch B has 184 boys and 201 girls,

   Branch C has 203 boys and 143 girls.

   Express the above information in matrix form.

   b. If there are 210 apples, 167 mangoes and 78 oranges in shop A and 198 apples, 200 mangoes and 103 oranges in shop B, represent this information in the form of a matrix.

9. a. For the given matrix $A = \begin{pmatrix} 1 & 4 & 3 & -7 \\ 3 & 6 & -4 & 8 \\ -2 & 7 & -1 & 2 \end{pmatrix}$,

   i. what is the order of the matrix? ii. find the values of a_{12}, a_{23} and a_{14}.

   iii. find the values of $a_{11} + a_{31}$ and $a_{24} - a_{14}$.

b. If $A = \begin{pmatrix} 2 & -3 & 7 \\ 3 & 8 & 4 \\ 6 & -7 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 4 & 1 \\ 6 & -5 & -3 \end{pmatrix}$,

i. write the orders of A and B.

ii. find the values of a_{22}, a_{31}, b_{12} and b_{23}.

iii. find the values of $a_{11} + b_{23}$, $a_{23} - b_{12}$.

10. State the types of the following matrices:

a. $\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$ b. $\begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$ c. $(2 \ \ 7)$ d. $\begin{pmatrix} 10 & 0 \\ 0 & -3 \end{pmatrix}$

e. $\begin{pmatrix} -4 & 0 \\ 0 & -4 \end{pmatrix}$ f. $\begin{pmatrix} 0 & -2 \\ -2 & 0 \end{pmatrix}$ g. $\begin{pmatrix} 2 & 0 \\ 3 & 7 \end{pmatrix}$ h. $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

i. $\begin{pmatrix} 1 & 2 & 5 \\ -6 & 7 & 3 \end{pmatrix}$ j. $\begin{pmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & -3 \end{pmatrix}$ k. $\begin{pmatrix} 1 & 2 & 3 \\ 2 & -2 & 4 \\ 3 & 4 & 1 \end{pmatrix}$

ℓ. $\begin{pmatrix} 2 & 4 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 4 \end{pmatrix}$ m. $\begin{pmatrix} -2 & 0 & 0 \\ 3 & 5 & 0 \\ 5 & 1 & 6 \end{pmatrix}$ n. $\begin{pmatrix} b & c & d \\ 0 & -a & e \\ 0 & 0 & -f \end{pmatrix}$

o. $\begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}$ p. $\begin{pmatrix} a & b & c \\ b & r & -d \\ c & -d & s \end{pmatrix}$

11. State which of the following matrices are equal.

$A = \begin{pmatrix} 1 & 3 \\ 7 & -5 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 2 \\ 3 & 7 \\ 1 & -4 \end{pmatrix}$, $C = \begin{pmatrix} 6 & 1 & 4 \\ 7 & 4 & 0 \end{pmatrix}$,

$D = \begin{pmatrix} 7 & 1 \\ 5 & 6 \end{pmatrix}$, $E = \begin{pmatrix} 1 & 3 \\ 7 & -5 \end{pmatrix}$, $F = \begin{pmatrix} 0 & 2 \\ 3 & 7 \\ 1 & -4 \end{pmatrix}$

12. Construct a 2×3 matrix whose elements a_{ij} are given by the formula.

a. $a_{ij} = i + 2j$ b. $a_{ij} = j$ c. $a_{ij} = 3i - 2j$

13. Construct a 3×3 matrix whose elements b_{ij} are given by the relation.

a. $b_{ij} = 2i + 3j$ b. $b_{ij} = 3j - i$ c. $b_{ij} = 4i - 3j$

14. a. If $\begin{pmatrix} 2 & x \\ 5 & -2 \end{pmatrix}$ and $\begin{pmatrix} y & -3 \\ 5 & -2 \end{pmatrix}$ are equal matrices, find the values of x and y.

b. If $\begin{pmatrix} 3 & a \\ b & 4 \end{pmatrix} = \begin{pmatrix} 3 & b+1 \\ 4 & 4 \end{pmatrix}$, find the values of a and b.

c. If $\begin{pmatrix} x+y & 3 \\ x & y \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ y+6 & -2 \end{pmatrix}$, find the alues of x and y.

d. If $\begin{pmatrix} a & a+b \\ -3 & c+6 \end{pmatrix} = \begin{pmatrix} 2 & 5 \\ c & 1+d \end{pmatrix}$, find the values of a, b, c and d.

e. If $\begin{pmatrix} 2p & q+1 \\ 5 & q-3 \end{pmatrix} = \begin{pmatrix} p+2 & r+s \\ r+1 & p+1 \end{pmatrix}$, find the values of p, q, r and s.

* * *

3.2 Operation of Matrices

Introduction

Here we deal with the following operations on matrices to express two or more matrices into a single matrix.

a. Addition of matrices
b. Subtraction of matrices
c. Multiplication of a matrix by a scalar
d. Multiplication of matrices
e. Transpose of a matrix

a. Addition of Two Matrices

The sum of two matrices A and B of same order is a new matrix whose each element is the sum of corresponding elements of A and B. It is denoted by A + B and the order of A + B is same as that of A or B.

Note: If two matrices are of different orders then their sum is not defined.

The following are the matrices of the number of boys and girls in three classes of two schools A and B.

$$\begin{array}{cc} & \textbf{Group A} \\ & \begin{array}{cc}\text{Boys} & \text{Girls}\end{array} \\ P = \begin{array}{c}\text{Class 10}\\ \text{Class 11}\\ \text{Class 12}\end{array} & \begin{pmatrix}25 & 30\\ 18 & 21\\ 16 & 18\end{pmatrix}\end{array} \qquad \begin{array}{cc} & \textbf{Group B} \\ & \begin{array}{cc}\text{Boys} & \text{Girls}\end{array} \\ Q = \begin{array}{c}\text{Class 10}\\ \text{Class 11}\\ \text{Class 12}\end{array} & \begin{pmatrix}18 & 10\\ 20 & 18\\ 10 & 12\end{pmatrix}\end{array}$$

Now, we want to find the total number of boys and girls classwise.

Total boys in class 10 = 25 + 18 = 43
Total girls in class 10 = 30 + 10 = 40
Total boys in class 11 = 18 + 20 = 38
Total girls in class 11 = 21 + 18 = 39
Total boys in class 12 = 16 + 10 = 26
Total girls in class 12 = 18 + 12 = 30

The above information can be obtained by adding given two matrices P and Q.

$$\begin{array}{cccc} & \textbf{Group A} & & \textbf{Group B} \\ & \begin{array}{cc}\text{Boys} & \text{Girls}\end{array} & & \begin{array}{cc}\text{Boys} & \text{Girls}\end{array} \\ P + Q = \begin{array}{c}\text{Class 10}\\ \text{Class 11}\\ \text{Class 12}\end{array} & \begin{pmatrix}25 & 30\\ 18 & 21\\ 16 & 18\end{pmatrix} & + & \begin{pmatrix}18 & 10\\ 20 & 18\\ 10 & 12\end{pmatrix}\end{array}$$

$$\begin{array}{cc} & \begin{array}{cc}\text{Boys} & \text{Girls}\end{array} \\ P + Q = \begin{array}{c}\text{Class 10}\\ \text{Class 11}\\ \text{Class 12}\end{array} & \begin{pmatrix}25 + 18 & 30 + 10\\ 18 + 20 & 21 + 18\\ 16 + 10 & 18 + 12\end{pmatrix}\end{array}$$

$$P+Q=\begin{array}{c} \\ \text{Class 10} \\ \text{Class 11} \\ \text{Class 12}\end{array}\begin{array}{c}\begin{array}{cc}\text{Boys} & \text{Girls}\end{array}\\ \begin{pmatrix} 43 & 40 \\ 38 & 39 \\ 26 & 30 \end{pmatrix}\end{array}$$

Example 1:

If $A=\begin{pmatrix} 1 & 3 & -4 \\ 7 & -3 & 0 \end{pmatrix}$ *and* $B=\begin{pmatrix} -5 & 3 & 0 \\ 2 & 4 & -4 \end{pmatrix}$, *find* $A+B$.

Here, $A=\begin{pmatrix} 1 & 3 & -4 \\ 7 & -3 & 0 \end{pmatrix}$ and $B=\begin{pmatrix} -5 & 3 & 0 \\ 2 & 4 & -4 \end{pmatrix}$

$$\text{Now, } A+B=\begin{pmatrix} 1 & 3 & -4 \\ 7 & -3 & 0 \end{pmatrix}+\begin{pmatrix} -5 & 3 & 0 \\ 2 & 4 & -4 \end{pmatrix}$$

$$\text{or,}\quad A+B=\begin{pmatrix} 1-5 & 3+3 & -4+0 \\ 7+2 & -3+4 & 0-4 \end{pmatrix}$$

$$\therefore\quad A+B=\begin{pmatrix} -4 & 6 & -4 \\ 9 & 1 & -4 \end{pmatrix}$$

b. Subtraction of Matrices

The subtraction of two matrices A and B of same order is a new matrix whose each element is obtained by subtracting the elements of B from corresponding elements of A. It is denoted by A – B and the order of A – B is same as that of A or B.

Note: If two matrices are of different orders then their difference is not defined.

Example 2:

If $P=\begin{pmatrix} 2 & -3 \\ 1 & 4 \\ 6 & 7 \end{pmatrix}$ *and* $Q=\begin{pmatrix} -2 & 7 \\ 0 & -5 \\ 3 & 8 \end{pmatrix}$, *find* $P-Q$ *and* $Q=P$.

Here, $P=\begin{pmatrix} 2 & -3 \\ 1 & 4 \\ 6 & 7 \end{pmatrix}$ and $Q=\begin{pmatrix} -2 & 7 \\ 0 & -5 \\ 3 & 8 \end{pmatrix}$

$$P-Q=\begin{pmatrix} 2 & -3 \\ 1 & 4 \\ 6 & 7 \end{pmatrix}-\begin{pmatrix} -2 & 7 \\ 0 & -5 \\ 3 & 8 \end{pmatrix}=\begin{pmatrix} 2+2 & -3-7 \\ 1-0 & 4+5 \\ 6-3 & 7-8 \end{pmatrix}=\begin{pmatrix} 4 & -10 \\ 1 & 9 \\ 3 & -1 \end{pmatrix}$$

Similarly,

$$Q-P=\begin{pmatrix} -2 & 7 \\ 0 & -5 \\ 3 & 8 \end{pmatrix}-\begin{pmatrix} 2 & -3 \\ 1 & 4 \\ 6 & 7 \end{pmatrix}$$

$$=\begin{pmatrix} -2-2 & 7+3 \\ 0-1 & -5-4 \\ 3-6 & 8-7 \end{pmatrix}=\begin{pmatrix} -4 & 10 \\ -1 & -9 \\ 3 & 1 \end{pmatrix}$$

Properties of Matrix Addition

Addition of matrices satisfy the following properties.

⇒ Closure Property

If A and B are two matrices of same order, then the sum A + B is also a matrix of the same order as that of A or B. For example:

$$A=\begin{pmatrix}2 & 3 & 1\\ 2 & -1 & 7\end{pmatrix}_{2\times3} \text{ and } B=\begin{pmatrix}4 & -7 & 0\\ -3 & -2 & 3\end{pmatrix}_{2\times3}, \text{ then}$$

$$\text{then } A+B=\begin{pmatrix}2 & 3 & 1\\ 2 & -1 & 7\end{pmatrix}+\begin{pmatrix}4 & -7 & 0\\ -3 & -2 & 3\end{pmatrix}=\begin{pmatrix}6 & -4 & 1\\ -1 & -3 & 10\end{pmatrix}_{2\times3}$$

Hence, the order of A + B is 2×3 that is same as the order of A or B.

⇒ Commutative Property

If A and B are two matrices of same order, then $A + B = B + A$.

For example:

$$A=\begin{pmatrix}1 & 3\\ -2 & 4\\ 0 & 5\end{pmatrix} \text{ and } B=\begin{pmatrix}-3 & 1\\ 5 & 3\\ -1 & 0\end{pmatrix}$$

$$A+B=\begin{pmatrix}1 & 3\\ -2 & 4\\ 0 & 5\end{pmatrix}+\begin{pmatrix}-3 & 1\\ 5 & 3\\ -1 & 0\end{pmatrix}=\begin{pmatrix}-2 & 4\\ 3 & 7\\ -1 & 5\end{pmatrix}$$

$$B+A=\begin{pmatrix}-3 & 1\\ 5 & 3\\ -1 & 0\end{pmatrix}+\begin{pmatrix}1 & 3\\ -2 & 4\\ 0 & 5\end{pmatrix}=\begin{pmatrix}-2 & 4\\ 3 & 7\\ -1 & 5\end{pmatrix}$$

$\therefore\quad A + B = B + A.$

⇒ Associative Property

If A, B and C are three matrices of same order, then $(A + B) + C = A + (B + C)$.

For example:

$$\text{If } A=\begin{pmatrix}1 & 4\\ -2 & 3\end{pmatrix}, B=\begin{pmatrix}0 & 2\\ -3 & 4\end{pmatrix} \text{ and } C=\begin{pmatrix}1 & 5\\ 6 & -1\end{pmatrix}$$

$$\text{then } A+B=\begin{pmatrix}1 & 4\\ -2 & 3\end{pmatrix}+\begin{pmatrix}0 & 2\\ -3 & 4\end{pmatrix}=\begin{pmatrix}1+0 & 4+2\\ -2-3 & 3+4\end{pmatrix}=\begin{pmatrix}1 & 6\\ -5 & 7\end{pmatrix}$$

$$\text{Now, } (A+B)+C=\begin{pmatrix}1 & 6\\ -5 & 7\end{pmatrix}+\begin{pmatrix}1 & 5\\ 6 & -1\end{pmatrix}$$

$$=\begin{pmatrix}1+1 & 6+5\\ -5+6 & 7-1\end{pmatrix}=\begin{pmatrix}2 & 11\\ 1 & 6\end{pmatrix}$$

$$\text{Again, } B + C = \begin{pmatrix} 0 & 2 \\ -3 & 4 \end{pmatrix} + \begin{pmatrix} 1 & 5 \\ 6 & -1 \end{pmatrix} = \begin{pmatrix} 0+1 & 2+5 \\ -3+6 & 4-1 \end{pmatrix} = \begin{pmatrix} 1 & 7 \\ 3 & 3 \end{pmatrix}$$

$$\text{Now, } A + (B + C) = \begin{pmatrix} 1 & 4 \\ -2 & 3 \end{pmatrix} + \begin{pmatrix} 1 & 7 \\ 3 & 3 \end{pmatrix} = \begin{pmatrix} 1+1 & 4+7 \\ -2+3 & 3+3 \end{pmatrix} = \begin{pmatrix} 2 & 11 \\ 1 & 6 \end{pmatrix}$$

$$\therefore \quad (A + B) + C = A + (B + C)$$

⇒ Additive Identity

If A is any matrix and O is a null matrix of same order, then $A + O = O + A = A$.

For example.

$$\text{If } A = \begin{pmatrix} 1 & 3 & 7 \\ -4 & 0 & -2 \end{pmatrix} \text{ and } O = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \text{ then}$$

$$A + O = \begin{pmatrix} 1 & 3 & 7 \\ -4 & 0 & -2 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1+0 & 3+0 & 7+0 \\ -4+0 & 0+0 & -2+0 \end{pmatrix} = \begin{pmatrix} 1 & 3 & 7 \\ -4 & 0 & -2 \end{pmatrix}$$

$$\text{Similarly, } O + A = \begin{pmatrix} 1 & 3 & 7 \\ -4 & 0 & -2 \end{pmatrix} = A$$

Hence, $A + O = O + A = A$.

⇒ Additive Inverse

If A is any matrix then there exists a negative matrix of A i.e. $-A$ such that $A + (-A) = (-A) + A = O$. For example,

$$\text{If } A = \begin{pmatrix} 3 & -4 \\ -1 & 5 \end{pmatrix} \text{ then } -A = \begin{pmatrix} -3 & 4 \\ 1 & -5 \end{pmatrix}$$

$$A + (-A) = \begin{pmatrix} 3 & -4 \\ -1 & 5 \end{pmatrix} + \begin{pmatrix} -3 & 4 \\ 1 & -5 \end{pmatrix} = \begin{pmatrix} 3-3 & -4+4 \\ -1+1 & 5-5 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = O$$

Similarly, $(-A) + A = O$

$\therefore\ A + (-A) = (-A) + A = O.$

c. Multiplication of Matrix by a Scalar

If A be a matrix of any order and k be a real number then the scalar multiplication of A by k is a new matrix obtained by multiplying each element of A by k and is denoted by kA.

$$\text{For example: If } A = \begin{pmatrix} 2 & -1 & 3 \\ 1 & 4 & -2 \end{pmatrix}, \text{ then by addition of matrices}$$

$$2A = A + A = \begin{pmatrix} 2 & -1 & 3 \\ 1 & 4 & -2 \end{pmatrix} + \begin{pmatrix} 2 & -1 & 3 \\ 1 & 4 & -2 \end{pmatrix}$$

$$= \begin{pmatrix} 2+2 & -1+(-1) & 3+3 \\ 1+1 & 4+4 & -2+(-2) \end{pmatrix} = \begin{pmatrix} 4 & -2 & 6 \\ 2 & 8 & -4 \end{pmatrix}$$

$$2A = \begin{pmatrix} 2 \times 2 & 2 \times -1 & 2 \times 3 \\ 2 \times 1 & 2 \times 4 & 2 \times (-2) \end{pmatrix}$$

$$2A = 2\begin{pmatrix} 2 & -1 & 3 \\ 1 & 4 & -2 \end{pmatrix}$$

Similarly, $\frac{1}{2}A = \frac{1}{2}\begin{pmatrix} 2 & -1 & 3 \\ 1 & 4 & -2 \end{pmatrix}$

$$= \begin{pmatrix} \frac{1}{2}\times 2 & \frac{1}{2}\times(-1) & \frac{1}{2}\times 3 \\ \frac{1}{2}\times 1 & \frac{1}{2}\times 4 & \frac{1}{2}\times(-2) \end{pmatrix} = \begin{pmatrix} 1 & -\frac{1}{2} & \frac{3}{2} \\ \frac{1}{2} & 2 & -1 \end{pmatrix}$$

and $-3A = -3\begin{pmatrix} 2 & -1 & 3 \\ 1 & 4 & -2 \end{pmatrix}$

$$= \begin{pmatrix} -3\times2 & -3\times(-1) & -3\times3 \\ -3\times1 & -3\times4 & -3\times(-2) \end{pmatrix} = \begin{pmatrix} -6 & 3 & -9 \\ -3 & -12 & 6 \end{pmatrix}.$$

Example 3:

If $A = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} -2 & 3 \\ -1 & 4 \end{pmatrix}$, find $2A + 3B$ and $A - 4B$.

Here, $A = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} -2 & 3 \\ -1 & 4 \end{pmatrix}$,

Now, $2A + 3B = 2\begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix} + 3\begin{pmatrix} -2 & 3 \\ -1 & 4 \end{pmatrix} = \begin{pmatrix} 4 & 2 \\ 6 & 8 \end{pmatrix} + \begin{pmatrix} -6 & 9 \\ -3 & 12 \end{pmatrix}$

$$= \begin{pmatrix} 4+(-6) & 2+9 \\ 6+(-3) & 8+12 \end{pmatrix} = \begin{pmatrix} -2 & 11 \\ 3 & 20 \end{pmatrix}$$

$$A - 4B = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix} - 4\begin{pmatrix} -2 & 3 \\ -1 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix} - \begin{pmatrix} -8 & 12 \\ -4 & 16 \end{pmatrix}$$

$$= \begin{pmatrix} 2-(-8) & 1-12 \\ 3-(-4) & 4-16 \end{pmatrix}$$

$$= \begin{pmatrix} 10 & -11 \\ 7 & -12 \end{pmatrix}$$

Example 4:

If $X + Y = \begin{pmatrix} -1 & 9 \\ 7 & 4 \end{pmatrix}$ and $2X - Y = \begin{pmatrix} 7 & 0 \\ 2 & 8 \end{pmatrix}$, find the matrices X and Y.

The given equations are

$$X + Y = \begin{pmatrix} -1 & 9 \\ 7 & 4 \end{pmatrix} \qquad \text{.........} \; (1)$$

$$2X - Y = \begin{pmatrix} 7 & 0 \\ 2 & 8 \end{pmatrix} \qquad \text{.........} \; (2)$$

Adding equations (1) and (2).

$$(X+Y)+(2X-Y)=\begin{pmatrix}-1 & 9\\ 7 & 4\end{pmatrix}+\begin{pmatrix}7 & 0\\ 2 & 8\end{pmatrix}$$

$$\text{or, } 3X=\begin{pmatrix}-1 & 9\\ 7 & 4\end{pmatrix}+\begin{pmatrix}7 & 0\\ 2 & 8\end{pmatrix}$$

$$\text{or, } 3X=\begin{pmatrix}-1+7 & 9+0\\ 7+2 & 4+8\end{pmatrix}$$

$$\text{or, } 3X=\begin{pmatrix}6 & 9\\ 9 & 12\end{pmatrix}$$

$$\text{or, } X=\frac{1}{3}\begin{pmatrix}6 & 9\\ 9 & 12\end{pmatrix}=\begin{pmatrix}2 & 3\\ 3 & 4\end{pmatrix}$$

Now from equation (1), we have

$$X+Y=\begin{pmatrix}-1 & 9\\ 7 & 4\end{pmatrix}$$

$$\text{or, } \begin{pmatrix}2 & 3\\ 3 & 4\end{pmatrix}+Y=\begin{pmatrix}-1 & 9\\ 7 & 4\end{pmatrix}$$

$$\text{or, } Y=\begin{pmatrix}-1 & 9\\ 7 & 4\end{pmatrix}-\begin{pmatrix}2 & 3\\ 3 & 4\end{pmatrix}$$

$$\text{or, } Y=\begin{pmatrix}-1-2 & 9-3\\ 7-3 & 4-4\end{pmatrix}$$

$$\therefore\ Y=\begin{pmatrix}-3 & 6\\ 4 & 0\end{pmatrix}$$

Example 5:

If $2\begin{pmatrix}x & 3\\ -1 & 4\end{pmatrix}+4\begin{pmatrix}1 & y\\ 1+x & -3\end{pmatrix}=\begin{pmatrix}2 & 10\\ z & y+w\end{pmatrix}$, ***find the values of w, x, y and z.***

Here, $2\begin{pmatrix}x & 3\\ -1 & 4\end{pmatrix}+4\begin{pmatrix}1 & y\\ 1+x & -3\end{pmatrix}=\begin{pmatrix}2 & 10\\ z & y+w\end{pmatrix}$

$$\text{or, } \begin{pmatrix}2x & 6\\ -2 & 8\end{pmatrix}+\begin{pmatrix}4 & 4y\\ 4+4x & -12\end{pmatrix}=\begin{pmatrix}2 & 10\\ z & y+w\end{pmatrix}$$

$$\text{or, } \begin{pmatrix}2x+4 & 6+4y\\ 2+4x & -4\end{pmatrix}=\begin{pmatrix}2 & 10\\ z & y+w\end{pmatrix}$$

or, $2x+4=2$, or, $6+4y=10$,

or, $2x=-2$, or, $4y=4$

$\therefore$ $x=-1$ $\therefore$ $y=1$

$2+4x=z$, $-4=y+w$

or, $2+4\times-1=z$ or, $-4=1+w$

$\therefore$ $z=-2$ $\therefore$ $w=-5$

$\therefore$ $w=-5$, $x=-1$, $y=1$ and $z=-2$.

EXERCISE : 3.2A

1. Find the values of a, b, c and d.

a. $\begin{pmatrix} 2 & -3 \\ a & 4 \end{pmatrix} = \begin{pmatrix} b & c \\ -4 & 4 \end{pmatrix}$

b. $\begin{pmatrix} 1 & a+2 & 3 \\ b-3 & 2 & 6 \end{pmatrix} = \begin{pmatrix} 1 & 5 & 5 \\ 2 & c+4 & 6 \end{pmatrix}$

c. $\begin{pmatrix} a+2 & b+2a \\ 7 & c-4 \end{pmatrix} = \begin{pmatrix} 3 & -3 \\ d-2 & d-7 \end{pmatrix}$

d. $\begin{pmatrix} a+b & a-b \\ d & 7 \end{pmatrix} = \begin{pmatrix} 4 & 2 \\ a+1 & c+b \end{pmatrix}$

2. Add the following matrices:

a. $\begin{pmatrix} 1 & 3 \\ -4 & 2 \end{pmatrix} + \begin{pmatrix} -2 & 1 \\ -7 & 2 \end{pmatrix}$

b. $\begin{pmatrix} 1 & 5 & 5 \\ 4 & 6 & -6 \end{pmatrix} + \begin{pmatrix} -2 & -1 & 0 \\ 0 & 2 & 4 \end{pmatrix}$

c. $\begin{pmatrix} 1 & -3 \\ 2 & 0 \\ 5 & -4 \end{pmatrix} + \begin{pmatrix} 1 & 4 \\ 2 & -3 \\ 0 & 7 \end{pmatrix}$

d. $\begin{pmatrix} 1 & 2 & -3 \\ 4 & -5 & 6 \\ -7 & 8 & 9 \end{pmatrix} + \begin{pmatrix} 1 & 2 & -3 \\ 4 & -5 & 6 \\ -7 & 8 & 9 \end{pmatrix}$

3. Subtract the following matrices:

a. $\begin{pmatrix} 2 \\ 3 \end{pmatrix} - \begin{pmatrix} -7 \\ 0 \end{pmatrix}$

b. $\begin{pmatrix} 1 & 2 & 7 \\ -7 & 4 & 0 \end{pmatrix} - \begin{pmatrix} 3 & 1 & 4 \\ -7 & 6 & 0 \end{pmatrix}$

c. $\begin{pmatrix} 3a & b \\ -3b & -2a \end{pmatrix} - \begin{pmatrix} 2a & -b \\ 2b & a \end{pmatrix}$

d. $\begin{pmatrix} 8 & 6 \\ 4 & 7 \\ 3 & -2 \end{pmatrix} - \begin{pmatrix} 10 & 0 \\ -2 & 8 \\ 3 & 5 \end{pmatrix}$

4. If $A = \begin{pmatrix} -2 & 3 \\ 4 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 8 & 8 \\ 10 & -6 \end{pmatrix}$, find:

a. $A+B$ b. $A-B$ c. $2A$ d. $-\frac{1}{2}B$

5. $A = \begin{pmatrix} 1 & 3 \\ 5 & 4 \end{pmatrix}$, $B = \begin{pmatrix} -2 & 1 \\ 0 & 3 \end{pmatrix}$ and $C = \begin{pmatrix} -3 & 1 \\ -1 & 4 \end{pmatrix}$, then find

a. $A+B+C$ b. $A+2B$ c. $\frac{1}{2}A-2B$ d. $A-B+2C$

e. $2A-\frac{1}{2}B+C$ f. $A-\frac{1}{3}B+\frac{1}{2}C$

6. a. Solve for the matrix X from equation $X + \begin{pmatrix} 1 & 3 \\ 5 & -3 \end{pmatrix} = \begin{pmatrix} 4 & 7 \\ 8 & 9 \end{pmatrix}$.

b. If $A = \begin{pmatrix} 1 & -2 \\ -3 & 4 \end{pmatrix}$, find the matrix X such that $A - 2X = \begin{pmatrix} 3 & 4 \\ 5 & 2 \end{pmatrix}$.

c. If $\begin{pmatrix} x \\ -3 \end{pmatrix} + \begin{pmatrix} 2 \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$, find the values of x and y.

d. If $\begin{pmatrix} 2 & 1 \\ x & 7 \end{pmatrix} + \begin{pmatrix} 1 & x+y \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 8 \\ 5 & 8 \end{pmatrix}$, find the values of x and y.

7. Find the additive inverse of the following matrices.

a. $\begin{pmatrix} 2 & -1 \\ -4 & 5 \end{pmatrix}$ b. $\begin{pmatrix} -2 & 1 \\ 3 & 4 \\ 6 & -7 \end{pmatrix}$ c. $\begin{pmatrix} 2 & 3 & 4 \\ 1 & 4 & -3 \end{pmatrix}$ d. $\begin{pmatrix} 1 & -4 & 0 \\ 2 & -5 & 7 \\ 3 & -6 & 9 \end{pmatrix}$

8. If $A = \begin{pmatrix} 1 & 3 \\ 0 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} -3 & 2 \\ 1 & -5 \end{pmatrix}$, then find:

a. $A - 2B$ b. $2A + B$ c. $2A - 3B$ d. $4A + 2B$

9. a. If $P + Q = \begin{pmatrix} 3 & 3 \\ 2 & 5 \end{pmatrix}$ and $3P - Q = \begin{pmatrix} -3 & 5 \\ 6 & 7 \end{pmatrix}$, find P and Q.

b. If $2A - B = \begin{pmatrix} 0 & 4 \\ 2 & 1 \end{pmatrix}$ and $A + 2B = \begin{pmatrix} 5 & 2 \\ 11 & 3 \end{pmatrix}$, find A and B.

10. Find the values of a, b and c from the following equations.

a. $2\begin{pmatrix} a & 3 \\ 1 & 0 \end{pmatrix} + 3\begin{pmatrix} 3 & 1 \\ 2 & b \end{pmatrix} = \begin{pmatrix} 8 & 9 \\ 8 & 9 \end{pmatrix}$ b. $3\begin{pmatrix} a & 4 \\ 0 & 3 \end{pmatrix} + 2\begin{pmatrix} 1 & b \\ -2 & -3 \end{pmatrix} = \begin{pmatrix} 5 & 11 \\ -2c & -3 \end{pmatrix}$

11. If $A = \begin{pmatrix} 1 & 2 & -3 \\ -2 & 0 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 2 & -1 \\ 2 & 4 & 6 \end{pmatrix}$ and $C = \begin{pmatrix} 2 & 3 & 1 \\ 5 & 6 & 3 \end{pmatrix}$ then find:

a. $2A + B - C$ b. $A + 2B + 3C$ c. $\frac{1}{2}A + B - \frac{1}{3}C$ d. $\frac{3}{2}A - B + 2C$

12. a. If $A = \begin{pmatrix} 1 & 3 \\ -4 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} -3 & 5 \\ 6 & 7 \end{pmatrix}$, then find a matrix X such that $A - 2B - 3X = \begin{pmatrix} 10 & 2 \\ 2 & 1 \end{pmatrix}$.

b. If $P = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$, $Q = \begin{pmatrix} 1 & 0 \\ -2 & 7 \end{pmatrix}$ and $R = \begin{pmatrix} 6 & 6 \\ 8 & 10 \end{pmatrix}$, then find a matrix Y such that $3P + Q + 2Y = R$.

13. a. If $\begin{pmatrix} 1 & a \\ -1 & -2 \end{pmatrix} + \begin{pmatrix} b & 2 \\ 3 & c \end{pmatrix} = \begin{pmatrix} 5 & 15 \\ 2d & 3 \end{pmatrix}$, then find the values of a, b, c and d.

b. If $\begin{pmatrix} x-2 & 5 \\ 2z & 7 \end{pmatrix} + \begin{pmatrix} 1 & 5 \\ -3 & y+2 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, then find the values of x, y and z.

c. If $\begin{pmatrix} x & 7 \\ y & z \end{pmatrix} + 2\begin{pmatrix} y & w \\ -x & 2w \end{pmatrix} = \begin{pmatrix} 3 & -5z+3 \\ 4 & 10 \end{pmatrix}$, find the values of w, x, y and z.

14. a. If $A + B = \begin{pmatrix} -1 & 9 \\ 8 & 4 \end{pmatrix}$ and $2A - B = \begin{pmatrix} 7 & 0 \\ 4 & 17 \end{pmatrix}$, find the matrices A and B.

b. If $X + 3Y = \begin{pmatrix} 1 & -1 \\ 15 & -4 \end{pmatrix}$ and $2X - 4Y = \begin{pmatrix} -8 & 3 \\ -10 & -8 \end{pmatrix}$, find the matrices X and Y.

c. If I is a unit matrix of order 2×2, find the matrix A such that $2A - 3I = \begin{pmatrix} -1 & 4 \\ 6 & 5 \end{pmatrix}$.

* * *

Multiplication of Matrices

Let us see the following two tables of certain information.

Table I

Families	Quantity in kg	
	Rice	Potato
X	14	2
Y	18	3

Table II

Families	Price in Rs. per kg	
	Poush	Magh
Rice	25	26
Potato	12	14

The table I shows the amount of rice and potato consumed by two families X and Y and table II shows the price of rice and potato per kg in two different months Poush and Magh.

How can we find the total expenditure of families X and Y for the months Poush and Magh? It can be shown in the following table.

Table III

Families	**Poush**	**Magh**
X	14 × 25 + 2 × 12	14 × 26 + 2 × 14
Y	18 × 25 + 3 × 12	18 × 26 + 3 × 14

Above multiplication operation can be presented in the matrix form as follows:

$$\begin{pmatrix} 14 & 2 \\ 18 & 3 \end{pmatrix}_{2\times2} + \begin{pmatrix} 25 & 26 \\ 12 & 14 \end{pmatrix}_{2\times2} = \begin{pmatrix} 14\times25+2\times12 & 14\times26+2\times14 \\ 18\times25+3\times12 & 18\times26+3\times14 \end{pmatrix}_{2\times2}$$

In above multiplication of matrices, the elements of the first row of the first matrix are treated with the elements of the first column of the second matrix. So the number of elements in the first row of the first matrix and the number of elements in the first column of the second matrix must be equal. i.e. number of columns of first matrix = number of rows of the second matrix. The multiplication of two matrices will be clear from following example.

If $A = \begin{pmatrix} 1 & 2 \\ -3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 & -3 \\ 4 & 0 & -5 \end{pmatrix}$, the product of two matrices A and B is denoted by AB and is calculated by the following process.

Here, $A = \begin{pmatrix} 1 & 2 \\ -3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 & -3 \\ 4 & 0 & -5 \end{pmatrix}$.

$$\begin{pmatrix} \boxed{1 \quad 2} \\ -3 \quad 4 \end{pmatrix} \begin{pmatrix} \boxed{\begin{matrix} 1 \\ 4 \end{matrix}} & \begin{matrix} 2 \\ 0 \end{matrix} & \begin{matrix} -3 \\ -5 \end{matrix} \end{pmatrix} = \begin{pmatrix} \boxed{1\times1+2\times4} & \cdots\cdots\cdots & \cdots\cdots\cdots \\ \cdots\cdots\cdots & \cdots\cdots\cdots & \cdots\cdots\cdots \end{pmatrix}$$

$$\begin{pmatrix} \boxed{1 \quad 2} \\ -3 \quad 4 \end{pmatrix} \begin{pmatrix} \begin{matrix} 1 \\ 4 \end{matrix} & \boxed{\begin{matrix} 2 \\ 0 \end{matrix}} & \begin{matrix} -3 \\ -5 \end{matrix} \end{pmatrix} = \begin{pmatrix} 1\times1+2\times4 & \boxed{1\times2+2\times0} & \cdots\cdots\cdots \\ \cdots\cdots\cdots & \cdots\cdots\cdots & \cdots\cdots\cdots \end{pmatrix}$$

$$\begin{pmatrix} \boxed{1 \quad 2} \\ -3 \quad 4 \end{pmatrix} \begin{pmatrix} \begin{matrix} 1 \\ 4 \end{matrix} & \begin{matrix} 2 \\ 0 \end{matrix} & \boxed{\begin{matrix} -3 \\ -5 \end{matrix}} \end{pmatrix} = \begin{pmatrix} 1\times1+2\times4 & 1\times2+2\times0 & \boxed{1\times(-3)+2\times(-5)} \\ \cdots\cdots\cdots & \cdots\cdots\cdots & \cdots\cdots\cdots \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 \\ \boxed{-3} & \boxed{4} \end{pmatrix}\begin{pmatrix} \boxed{1} & 2 & -3 \\ \boxed{4} & 0 & -5 \end{pmatrix} = \begin{pmatrix} 1\times1+2\times4 & 1\times2+2\times0 & 1\times(-3)+2\times(-5) \\ \boxed{-3\times1+4\times4} & \dots\dots\dots & \dots\dots\dots \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 \\ \boxed{-3} & \boxed{4} \end{pmatrix}\begin{pmatrix} 1 & \boxed{2} & -3 \\ 4 & \boxed{0} & -5 \end{pmatrix} = \begin{pmatrix} 1\times1+2\times4 & 1\times2+2\times0 & 1\times(-3)+2\times(-5) \\ -3\times1+4\times4 & \boxed{-3\times2+4\times0} & \dots\dots\dots \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 \\ \boxed{-3} & \boxed{4} \end{pmatrix}\begin{pmatrix} 1 & 2 & \boxed{-3} \\ 4 & 0 & \boxed{-5} \end{pmatrix} = \begin{pmatrix} 1\times1+2\times4 & 1\times2+2\times0 & 1\times(-3)+2\times(-5) \\ -3\times1+4\times4 & -3\times2+4\times0 & \boxed{-3\times(-3)+4\times(-5)} \end{pmatrix}$$

$$\therefore AB = \begin{pmatrix} 1 & 2 \\ -3 & 4 \end{pmatrix}_{2\times\boxed{2}}\begin{pmatrix} 1 & 2 & -3 \\ 4 & 0 & -5 \end{pmatrix}_{\boxed{2}\times3}$$

$$= \begin{pmatrix} 1\times1+2\times4 & 1\times2+2\times0 & 1\times(-3)+2\times(-5) \\ -3\times1+4\times4 & -3\times2+4\times0 & -3\times(-3)+4\times(-5) \end{pmatrix}_{2\times3}$$

$$= \begin{pmatrix} 9 & 2 & -13 \\ 13 & -6 & -11 \end{pmatrix}_{2\times3}$$

Hence, if A and B are two matrices of order $m\times\boxed{n}$ and $\boxed{n}\times p$ respectively then the product AB is a matrix of order $m\times p$.

Properties of Matrix Multiplication

If A, B and C are the matrices which are comfortable for multiplication with each other. The matrices B and C are of same order then the following properties hold for matrix multiplication.

a. In general, $AB \neq BA$ (Non commutative)

b. $A(BC) = (AB)C$ (Associative property)

c. $A(B + C) = AB + AC$ (Distributive property)

d. $AI = IA = A$ (Multiplicative identity)

Where I is a unit matrix comfortable for AI and IA. The above properties can easily be verified.

Example 1:

Find the product of following matrices.

a. $\begin{pmatrix} 1 & 3 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ -2 \\ -3 \end{pmatrix}$ b. $\begin{pmatrix} 1 \\ 3 \end{pmatrix}\begin{pmatrix} 1 & 5 \end{pmatrix}$

a. $$\begin{pmatrix} 1 & 3 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ -2 \\ -3 \end{pmatrix} = \begin{pmatrix} 1\times1+3\times(-2)+2\times(-3) \end{pmatrix}$$

$$= (1-6-6) = (-11)$$

b. $$\begin{pmatrix} 1 \\ 3 \end{pmatrix}_{2\times1}\begin{pmatrix} 1 & 5 \end{pmatrix}_{1\times2} = \begin{pmatrix} 1\times1 & 1\times5 \\ 3\times1 & 3\times5 \end{pmatrix}_{2\times2}$$

$$= \begin{pmatrix} 1 & 5 \\ 3 & 15 \end{pmatrix}_{2\times2}$$

Example 2:

If $A = \begin{pmatrix} 1 & -4 \\ 7 & 3 \end{pmatrix}$ ***and*** $B = \begin{pmatrix} -1 & 2 \\ 3 & 0 \end{pmatrix}$***, find AB. Does BA define? If BA is defined, find BA and show that*** $AB \neq BA$***.***

Here, $A = \begin{pmatrix} 1 & -4 \\ 7 & 3 \end{pmatrix}_{2\times 2}$ and $B = \begin{pmatrix} -1 & 2 \\ 3 & 0 \end{pmatrix}_{2\times 2}$

$$AB = \begin{pmatrix} 1 & -4 \\ 7 & 3 \end{pmatrix}\begin{pmatrix} -1 & 2 \\ 3 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1\times -1 + (-4)\times 3 & 1\times 2 + (-4)\times 0 \\ 7\times -1 + 3\times 3 & 7\times 2 + 3\times 0 \end{pmatrix} = \begin{pmatrix} -13 & 2 \\ 2 & 14 \end{pmatrix} \text{ and}$$

$$BA = \begin{pmatrix} -1 & 2 \\ 3 & 0 \end{pmatrix}\begin{pmatrix} 1 & -4 \\ 7 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} -1\times 1 + 2\times 7 & -1\times -4 + 2\times 3 \\ 7\times -1 + 3\times 3 & 3\times -4 + 0\times 3 \end{pmatrix} = \begin{pmatrix} 13 & 10 \\ 3 & -12 \end{pmatrix}$$

Since, Corresponding elements of AB and BA are not equal, $AB \neq BA$.

Example 3:

If $A = \begin{pmatrix} 1 & 0 \\ 2 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 \\ -2 & 3 \end{pmatrix}$ ***and*** $C = \begin{pmatrix} 2 & 0 \\ 3 & -1 \end{pmatrix}$ ***verify that:***

a. $A(B + C) = AB + AC$ ***b.*** $A(BC) = (AB)C$

a. $B + C = \begin{pmatrix} 1 & 2 \\ -2 & 3 \end{pmatrix} + \begin{pmatrix} 2 & 0 \\ 3 & -1 \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}$

$$A(B + C) = \begin{pmatrix} 1 & 0 \\ 2 & -1 \end{pmatrix} \times \begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 3+0 & 2+0 \\ 6-1 & 4-2 \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ 5 & 2 \end{pmatrix}$$

$$AB = \begin{pmatrix} 1 & 0 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ -2 & 3 \end{pmatrix} = \begin{pmatrix} 1-0 & 2+0 \\ 2+2 & 4-3 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 4 & 1 \end{pmatrix}$$

$$AC = \begin{pmatrix} 1 & 0 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 3 & -1 \end{pmatrix} = \begin{pmatrix} 2+0 & 0+0 \\ 4-3 & 0+1 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix}$$

$\therefore$ $AB + AC = \begin{pmatrix} 1 & 2 \\ 4 & 1 \end{pmatrix} + \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ 5 & 2 \end{pmatrix}$

$\therefore$ $A(B + C) = AB + AC$.

b. $BC = \begin{pmatrix} 1 & 2 \\ -2 & 3 \end{pmatrix} \times \begin{pmatrix} 2 & 0 \\ 3 & -1 \end{pmatrix} = \begin{pmatrix} 2+6 & 0-0 \\ -4+9 & 0-3 \end{pmatrix} = \begin{pmatrix} 8 & -2 \\ 5 & -3 \end{pmatrix}$

$$A(BC) = \begin{pmatrix} 1 & 0 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} 8 & -2 \\ 5 & -3 \end{pmatrix} = \begin{pmatrix} 8+0 & -2+0 \\ 16-5 & -4+3 \end{pmatrix} = \begin{pmatrix} 8 & -2 \\ 11 & -1 \end{pmatrix}$$

$$AB = \begin{pmatrix} 1 & 0 \\ 2 & -1 \end{pmatrix} \times \begin{pmatrix} 1 & 2 \\ -2 & 3 \end{pmatrix} = \begin{pmatrix} 1+0 & 2-0 \\ 2+2 & 4-3 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 4 & 1 \end{pmatrix}$$

$$(AB)C = \begin{pmatrix} 1 & 2 \\ 4 & 1 \end{pmatrix} \times \begin{pmatrix} 2 & 0 \\ 3 & -1 \end{pmatrix} = \begin{pmatrix} 2+6 & 0-2 \\ 8+3 & 0-1 \end{pmatrix} = \begin{pmatrix} 8 & -2 \\ 11 & -1 \end{pmatrix}$$

$\therefore$ $A(BC) = (AB)C$.

Example 4:

If $A=\begin{pmatrix}1 & 2\\ 3 & 1\end{pmatrix}$ *show that* $A^2-2A-5I=O$*, where I and O are idendity and null matrices of order 2×2.*

✍ Here, $A=\begin{pmatrix}1 & 2\\ 3 & 1\end{pmatrix}, I=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$ and $O=\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}$, then

$$A^2=\begin{pmatrix}1 & 2\\ 3 & 1\end{pmatrix}\begin{pmatrix}1 & 2\\ 3 & 1\end{pmatrix}=\begin{pmatrix}1\times1+2\times3 & 1\times2+2\times1\\ 3\times1+1\times3 & 3\times2+1\times1\end{pmatrix}$$

$$=\begin{pmatrix}1+6 & 2+2\\ 3+3 & 6+1\end{pmatrix}=\begin{pmatrix}7 & 4\\ 6 & 7\end{pmatrix}$$

Now, $A^2-2A-5I=\begin{pmatrix}7 & 4\\ 6 & 7\end{pmatrix}-2\times\begin{pmatrix}1 & 2\\ 3 & 1\end{pmatrix}-5\times\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

$$=\begin{pmatrix}7 & 4\\ 6 & 7\end{pmatrix}-\begin{pmatrix}2 & 4\\ 6 & 2\end{pmatrix}-\begin{pmatrix}5 & 0\\ 0 & 5\end{pmatrix}$$

$$=\begin{pmatrix}7-2-5 & 4-4-0\\ 6-6-0 & 7-2-5\end{pmatrix}=\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}$$ proved.

Example 5:

If $A=\begin{pmatrix}1 & 2\\ -3 & 0\end{pmatrix}$*, find a matrix X such that* $AX=\begin{pmatrix}5 & 6\\ -3 & 0\end{pmatrix}$.

✍ A is a 2×2 matrix and the matrix on the right hand side is also a 2×2 matrix, so X must be a 2×2 matrix.

Let $X=\begin{pmatrix}a & b\\ c & d\end{pmatrix}$ and $AX=\begin{pmatrix}5 & 6\\ -3 & 0\end{pmatrix}$

or, $\begin{pmatrix}1 & 2\\ -3 & 0\end{pmatrix}\begin{pmatrix}a & b\\ c & d\end{pmatrix}=\begin{pmatrix}5 & 6\\ -3 & 0\end{pmatrix}$

or, $\begin{pmatrix}a+2c & b+2d\\ -3a+0 & -3b+0\end{pmatrix}=\begin{pmatrix}5 & 6\\ -3 & 0\end{pmatrix}$

or, $-3a=-3$ $\quad\therefore a=1$

or, $a+2c=5$ $\quad\therefore c=2$

or, $-3b=0$ $\quad\therefore b=0$

or, $b+2d=6$ $\quad\therefore d=3$

$\therefore\ X=\begin{pmatrix}a & b\\ c & d\end{pmatrix}=\begin{pmatrix}1 & 0\\ 2 & 3\end{pmatrix}$

Example 6:

If $\begin{pmatrix}7x+1\\ 6y-2\end{pmatrix}=\begin{pmatrix}2 & 3\\ -1 & 8\end{pmatrix}\begin{pmatrix}0\\ 2\end{pmatrix}$*, find the values of x and y.*

✍ We have, $\begin{pmatrix}7x+1\\ 6y-2\end{pmatrix}=\begin{pmatrix}2 & 3\\ -1 & 8\end{pmatrix}\begin{pmatrix}0\\ 2\end{pmatrix}$

or, $\begin{pmatrix}7x+1\\ 6y-2\end{pmatrix}=\begin{pmatrix}2\times0+3\times2\\ -1\times0+8\times2\end{pmatrix}$

or, $\begin{pmatrix}7x+1\\ 6y-2\end{pmatrix}=\begin{pmatrix}6\\ 16\end{pmatrix}$

Now, comparing corresponding elements, we have

$7x + 1 = 6$ and
or, $7x = 6 - 1$
or, $7x = 5$
$\therefore$ $x = \frac{5}{7}$

$6y - 2 = 16$
or, $6y = 16 + 2$
or, $6y = 18$
or, $y = \frac{18}{6} = 3$
$\therefore$ $y = 3$

Hence, $x = \frac{5}{7}$ and $y = 3$

Transpose of a Matrix

A matrix obtained by interchanging the rows and columns of a given matrix is known as transpose of the matrix. If A is given matrix its transpose is denoted by A^T or A'. For example if $A = \begin{pmatrix} 1 & 0 \\ 3 & -2 \\ 5 & 4 \end{pmatrix}$ then its transpose is $A^T = \begin{pmatrix} 1 & 3 & 5 \\ 0 & -2 & 4 \end{pmatrix}$.

Example 7:

If $A = \begin{pmatrix} 2 & 3 \\ -1 & 4 \end{pmatrix}$ *and* $B = \begin{pmatrix} 0 & -1 \\ 2 & 3 \end{pmatrix}$ *then prove that* $(AB)^T = B^T A^T$.

✍ Here, $A = \begin{pmatrix} 2 & 3 \\ -1 & 4 \end{pmatrix}$ then $A^T = \begin{pmatrix} 2 & -1 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & -1 \\ 2 & 3 \end{pmatrix}$ then $B^T = \begin{pmatrix} 0 & 2 \\ -1 & 3 \end{pmatrix}$.

Now, $AB = \begin{pmatrix} 2 & 3 \\ -1 & 4 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 0+6 & -2+9 \\ 0+8 & 1+12 \end{pmatrix} = \begin{pmatrix} 6 & 7 \\ 8 & 13 \end{pmatrix}$ and $(AB)^T = \begin{pmatrix} 6 & 8 \\ 7 & 13 \end{pmatrix}$

Again $B^T A^T = \begin{pmatrix} 0 & 2 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 0+6 & 0+8 \\ -2+9 & 1+12 \end{pmatrix} = \begin{pmatrix} 6 & 8 \\ 7 & 13 \end{pmatrix}$

$\therefore$ $(AB)^T = B^T A^T$.

EXERCISE : 3.2B

1. Find the product of the following matrices.

a. $(2\ \ 3)\begin{pmatrix} 4 \\ 5 \end{pmatrix}$

b. $(2\ \ 3\ \ 4)\begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$

c. $\begin{pmatrix} 0 & 2 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 4 \\ 5 \end{pmatrix}$

d. $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$

2. a. If $A = \begin{pmatrix} 2 & 0 & 3 \\ 1 & 4 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 \\ 2 & 4 \\ 3 & -1 \end{pmatrix}$, find AB and BA.

b. If $A = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} a & b \\ 0 & 2 \end{pmatrix}$ and $AB = \begin{pmatrix} 4 & 8 \\ 2 & 9 \end{pmatrix}$, find the values of a and b.

3. Find the value of x:

a. $(x\ \ -3)\begin{pmatrix} 2 \\ 4 \end{pmatrix} = (4)$

b. $(3\ \ x)\begin{pmatrix} 2x \\ 4 \end{pmatrix} = (20)$

c. $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} x+1 \\ y-2 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

d. $\begin{pmatrix} 2 & -1 \\ 5 & -2 \end{pmatrix}\begin{pmatrix} -3 & x \\ y & -4 \end{pmatrix} = \begin{pmatrix} -8 & 6 \\ -19 & 13 \end{pmatrix}$

e. $\begin{pmatrix} 2 & a \\ b & 4 \end{pmatrix}\begin{pmatrix} -2 \\ 5 \end{pmatrix} = \begin{pmatrix} 6 \\ 0 \end{pmatrix}$

f. $\begin{pmatrix} -2 & 0 \\ 0 & -5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ -10 \end{pmatrix}$

4. a. If $A = \begin{pmatrix} 3 & 4 \\ 2 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & -4 \\ -2 & 3 \end{pmatrix}$, then prove that AB is an identity matrix.

b. If $P = \begin{pmatrix} 2m & 7 \\ 5 & 9 \end{pmatrix}$, $Q = \begin{pmatrix} 9 & n \\ -5 & 4 \end{pmatrix}$ and PQ is an identity matrix, then find the values of m and n.

5. If $P = \begin{pmatrix} 1 & 3 \\ 1 & 4 \end{pmatrix}$ and $Q = \begin{pmatrix} 1 & 4 \\ 0 & -2 \\ 3 & 5 \end{pmatrix}$, find

a. P^T b. Q^T c. PQ^T

d. P^TQ^T e. $(PQ^T)^T$ f. $(QP)^T$

6. If $A = \begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 5 & -2 \\ 0 & 3 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 3 \\ 6 & -2 \end{pmatrix}$, then show that:

a. $AB \neq BA$ b. $(AB)C = A(BC)$

c. $A(B + C) = AB + AC$ d. $B(C - A) = BC - BA$

7. a. If $A = \begin{pmatrix} 4 & 2 & 1 \\ 3 & 4 & -5 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -2 \\ 0 & 3 \\ 1 & -4 \end{pmatrix}$, find AB. Does BA define? If BA is defined, find BA. Are AB and BA equal?

b. If $A = \begin{pmatrix} 0 & 4 \\ 0 & 6 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 4 \\ 3 & -5 \end{pmatrix}$ and $C = \begin{pmatrix} -2 & 5 \\ 3 & -5 \end{pmatrix}$, then show that AB = AC.

8. In each of the following cases, prove that AB = BA = I.

a. $A = \begin{pmatrix} 7 & 4 \\ 5 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & -4 \\ -5 & 7 \end{pmatrix}$ b. $A = \begin{pmatrix} 11 & -4 \\ -8 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 4 \\ 8 & 11 \end{pmatrix}$

9. Solve the matrix equations if X is a matrix of order 2×1.

a. $\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix} X = \begin{pmatrix} -4 \\ -7 \end{pmatrix}$ b. $\begin{pmatrix} -4 & 0 \\ 5 & 7 \end{pmatrix} X = \begin{pmatrix} 4 \\ 9 \end{pmatrix}$

10. a. Which matrix pre-multiplies to the matrix $\begin{pmatrix} 1 & 1 \\ 3 & 4 \end{pmatrix}$ give matrix $\begin{pmatrix} 4 & 6 \\ 5 & 2 \end{pmatrix}$?

b. Which matrix pre-multiplies to the matrix $\begin{pmatrix} 1 & -2 \\ 2 & 3 \end{pmatrix}$ to get the matrix $(4 \ -1)$?

11. If $A = \begin{pmatrix} 1 & 3 \\ 5 & 6 \end{pmatrix}$ and $B = \begin{pmatrix} -2 & -3 \\ 0 & 4 \end{pmatrix}$, show that:

a. $(A^T)^T = A$ b. $(A + B)^T = A^T + B^T$ c. $(AB)^T = B^TA^T$ d. $(2A)^T = 2A^T$

12. a. If $A = \begin{pmatrix} 4 & 2 \\ -1 & 1 \end{pmatrix}$, find A^2 and show that $A^2 - 5A + 6I = O$, where I and O are the identity and the zero matrices of order 2×2.

b. If $B = \begin{pmatrix} 2 & 2 \\ 1 & 2 \end{pmatrix}$, find B^2 and show that $B^2 - 5B + 8I = O$, where I and O are the identity and the zero matrices of order 2×2.

c. If $A = \begin{pmatrix} 1 & 4 \\ -2 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} -2 & 4 \\ -2 & 0 \end{pmatrix}$ then show that $A^2 - 8B - 17I = O$ where I and O are unit and zero matrices of order 2×2.

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